A TASTER OF SIMPLE AERODYNAMICS.

Mike Disney, (originally 2004.)

Contains just enough information for anyone who studied Physics for a couple of years at school to check out the Wright Brothers case, and also to see why tiny birds can fly non-stop across oceans.

TAKE OFF POWER REQUIRED

The lifting force F_L of a wing of area A moving at speed V will be:

$F_L \propto \rho A V^2$ where ρ is air density and $\propto$ means "proportional to"

The square in V^2 appears because as the wing moves faster it hits more air per second *and* hits it faster (i.e. imparts more momentum to it)[1]. The constant of proportionality is conventionality written as $C_L/2$ where C_L is a pure number (with a value around 1) called 'the Lift Coefficient'. Thus

$$F_L = \frac{1}{2} C_L \rho A V^2 \qquad (1)$$

For a flyer of mass M to take off the lift must exceed the downward force of gravity Mg (g is the downward force per unit mass, in other words the downward acceleration according to Newton). And so:

$\frac{1}{2} C_L \rho A V^2 > Mg$ where > means "more than'. So

$$V > V_{\min} = \left[\frac{2Mg}{C_L \rho A}\right]^{1/2} \qquad (2)$$

where as usual the exponent ½ means 'square root of' the bracketed quantity. ($V_{\min}$ is also called the stalling speed).

Your plane accelerates down the runway until it exceeds speed $V_{\min}$ when the wings develop enough lift to exceed the plane's weight Mg . Note an important insight here. Since the volume of any body (and hence its mass M) will scale with the cube of its length L, while areas such as A only scale with L squared, the right hand side of (2) scales with $L^{1/2}$ so that, all other things being equal, larger flyers must have higher

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[1] $\rho A V^2$ is the only simple combination of air-density, Wing area and Velocity which yields a force. That's one cunning way to see how this formula arises.

take-off speeds and cruising speeds (in proportion to the square roots of their relative sizes). A warbler can leap straight into the air, a Jumbo has to reach 180 mph first.

The speeding wing also develops a backward drag-force F_D, and just like F_L:

$$F_D = \frac{1}{2} C_D \rho A V^2 \qquad (3)$$

where this time C_D is its 'drag-coefficient' a pure number with a typical value of about 0.1 at slow speeds.

The drag force at take-off can be calculated from (3) by replacing the V using (2) when

$$F_D(\min) = \frac{1}{2} C_D \rho A \times \frac{2Mg}{C_L \rho A} = \frac{C_D}{C_L} Mg \qquad (4)$$

and as wind-tunnel measurements show that C_D / C_L is typically 0.2 at low speeds the drag then is about one fifth of the weight.

Unless the engine can deliver enough power to overcome this drag the flyer will never take off. By definition Power P = Force times speed so:

$$P > \frac{1}{2} C_D \rho A V^2 \times V$$

And if we replace V by the minimum take-off speed in (2) and use (4):

$$P(engine) > F_D(\min) \times V_{\min} = \frac{Mg}{C_L / C_D} \times \left(\frac{2Mg}{C_L \rho A} \right)^{1/2} \qquad (5)$$

which is the vital formula used to calculate the Wrights' required engine-power. If we put the known, or subsequently measured wind-tunnel values in (M=340 kg; A= 47.4 sq. meters; ρ =1.2 kg/cubic meter; C_L= 0.6; C_D= 0.12) we find:

V(min) = 14 m/sec = 32 mph

And $$P(engine) = \frac{1}{\eta} \times 9.5kW = \frac{1}{\eta} \times 13HP$$

Where η (eta) is the propeller-efficiency, the fraction of engine-power actually delivered as thrust. The Wright's twin contra-rotating pusher-propellers were rather crude and no one has suggested an efficiency better than 0.7, and if we further take into account the long (crossed) chains running inside pipes that were needed to transmit that power from the engine to the two props η would be more like 0.6. And what about ground drag? They had no wheels, only a clumsy wooden skid running along a wooden mono-rail. One should probably add another 10% needed for that, so altogether I reckon they needed an absolute minimum of 18 HP, and a more likely 24 HP to take off, as compared to the measured 12 HP available. By comparison Bleriot, with a lighter, much sleeker aeroplane, which included a wheeled undercarriage and a rather sophisticated carved propeller (see the photographs) needed 25 HP from his Anzani engine. [N.B. We have omitted the so called 'Induced Drag' which is particularly severe for a slow speed bi-plane like the Flyer; that could easily add another 50% to the take-off power the Wrights needed.]

MAXIMUM RANGE

Now for the "Christopher Problem", the problem of how far an animal like a bird can fly without refuelling itself. When a tiny warbler landed on our ship in mid Atlantic I became utterly intrigued by this problem, which took me ten years to solve.

If E is the energy of all the fuel stored aboard, the maximum range R(max) of any flying mechanism will be

$$R = V \times Endurance = V \times \frac{E}{P} = V \times \frac{E}{VF_D(\min)}$$

$$= \frac{E}{\left(C_D / C_L\right) Mg} = \frac{C_L}{C_D} \times \frac{\left(E / M\right)}{g} \qquad (6)$$

Equation (6), originally due to Louis Bregeut, but which took me 10 years to work out for myself, is absolutely fascinating because $\left(E / M\right)$, the amount of energy stored per unit-mass of the flyer, can be independent of size, as are all the factors in the equation. Thus *the range of a flyer will be independent of size*!!! If a Jumbo could cross the Atlantic, then so could a small bird with the same C_L / C_D *if* they both carried the same *proportion* of the same fuel, and burned it at the same efficiency. Birds and airliners carry about half their body-mass as fuel before taking off on long flights. Animal-fat (9 kilo-Calories per gram) is about 5 times as energy-intensive as aviation fuel, but animals, because of their relatively low temperatures, have a thermodynamic efficiency about one fifth of a jet engine (which is much hotter) so the two factors roughly cancel out meaning that even small creatures can migrate enormous distances. Since a warbler has a C_L / C_D of about 5, a Jumbo of about 15, I would expect a warbler to have a range of one third of a Jumbo, or about 2500 miles. By flying at slower and more efficient speeds small flyers can make up for their small sizes. [Swans are about the limit when it comes to birds; weighing in at 11 kilos their take off speed is so high (45 mph plus) they can barely manage it, and therefore they have to fly fast, and thus inefficiently].

Footnote: The pure numbers C_L, C_D and C_L / C_D conceal many of the complexities of aerodynamics. For a given wing you can measure them in a wind-tunnel, where they vary significantly only with 'the angle of attack' of the wing-section to the airflow. This was done retrospectively for the Wright Flyer when it was found $C_L = 0.6$ and $C_L / C_D = 5$. However the C_L / C_D vital for minimum-power and for maximum range [equations (4) and (6)] can be estimated from the observed angle of descent of a flyer when its power is turned off. In that case it will lose height at a vertical velocity U such that it its rate of loss of potential energy UMg balances the rate it must do work against drag = VF_D so:

$$UMg = V_{\min} F_D(\min) = V_{\min} \frac{C_L}{C_D} Mg$$

$$or \quad \text{U/V(i.e. angle of descent)} = \frac{C_D}{C_L} \qquad (7)$$

V/U for a warbler is about 5, for a seagull about 12, for an albatross about 24 and for my glider about 40. No wonder albatrosses can fly thousands of miles in between feeding their chicks. In my country (Wales) Shearwaters fly all the way to Portugal (nearly a thousand miles) to fetch back sardines for their young. The Northern Wheatear flies from Alaska 15,000 miles via Asia to winter in South Africa; its co-population in Eastern Canada flies the shorter (7000 mile) rout via Europe. Every year.

PSS: SAFETY MARGINS ON TAKEOFF (added 2021)

Anyone who has flown on a commercial jet airliner will know that it puts out by far the most power at take-off and just after, as it climbs steeply away, before throttling right back. Why? And what bearing has that on the Wright brothers' claim to have made the first powered flight?

An aircraft in the air but close to the ground could be in peril if its engines fail. To be reasonably safe it must either be able to nose down and glide to a safe landing on the remaining runway ahead, or else be high enough to turn back, glide downwind, then reverse itself again and come back in to land on the runway but with all engines out. Trainee pilots must train to deal with such emergencies as matter of course.

The secret of safety in all cases is raw engine power, the power to get up high enough before you run out of runway and so be able to turn back , make a complete circuit before re-settling back to the ground again with time and height to spare.

But how much raw power? It's easy to work out a rather simple formula:

$$\frac{P_{max}}{P_0} - 1 \geq S\gamma \frac{X}{X-X_0} \qquad (8)$$

Where:

P_{max} is the safe power you need.

$\geq$ means 'more than or equal to'.

P_0 is the minimum power needed to keep the plane in level flight [see (2) above]

X is the total length of runway

X_0 is the length of it required to take off normally.

While S and γ are desirable safety factors: γX is the total distance the engine- out plane may have to glide to make a circuit back (~ 3X) before it can resettle back on the runway in the required place and position., while S (more than 1) grants the pilot an extra safe margin of time beyond the minimum necessary to make a prudent engine-out landing. Thus with S= 1.5 he will have 50% more time than he actually needs.

If you try Equation (8) out under two extremes – a massive Jumbo, and a glider being launched by winch, it makes predictions which accurately mirror the facts. And if you try to squeeze all the safety factors to the very minimum you will find that for the Wrights then (8) :

P_{max}/P_{min} has to be more than 3.5.

So they needed not 12-18 HP to take off but 40 - 60 HP – vastly beyond the capabilities of their engine. So we are not talking about debatable margins here. They could never have taken off without a catapult. Their's wasn't powered flight. (NB. If we apply identical arguments to Bleriot's plane it has a safety margin of about 5: he could take off safely.)

N.B Incidentally this all raises an absolutely fascinating question. How did giant pterosaurs weighing up to 100 kilograms or more manage to fly in prehistoric times when they didn't even have feathers? All the aerodynamics above is totally against them. They must have had a secret; a very valuable secret. I think I know what it was, and if I'm right that could solve the global warming problem, and much else besides. You can follow up the story if you want to.[2]

In order to be brief this account is necessarily *highly simplified*. If you want to know more the four best books I know of on low speed flight are:
1 *The Simple Science of Flight*, Henk Tennekes, MIT Press, 1992. Very readable, but the last chapter on the Boeing 747 is wrong.
2 *Animal Flight*, 1972, Colin J Pennycuick, Edward Arnold UK, a very short book (less than 70 p) by *the* expert. He flew alongside vultures in a motor-glider to see how they did it.
3 *Bird Flight* by S Dhawan, 1991, Printed by the Indian Academy of Sciences. Dhawan, who I knew, was India's leading Space Scientist who taught us much by using the very high speed cameras developed to monitor rocket-launches to study the birds which fascinated him. For instance you can see that their wings flap and rotate mainly to propel air backwards (like a propeller) not downwards, as I had always assumed.
4 *More with Less*, by Paul Ciotti, Encounter Press San Francisco, 2002, is the biography of the great American aeronaut Paul MacReady, who built the aircraft which first crossed the Channel using human pedal power, then sunlight. A delightful book about a delightful man.

[Whatever you do though *don't* read the all-too-many books on aerodynamics written by mathematicians. Why? Because mathematics is not about cause and effect, but about logical consistency. Such books therefore contain statements akin to "That man can walk because he leaves footsteps behind." Not helpful.]

And if you are interested in bird migration read the quite beautiful *"The Island"* by R.M. Lockley (Penguin Books, 1980). I've read it at least a dozen times: about a young family who go out to live on a remote Welsh island in the 1930s and began to wonder about the other creatures who were their only companions. Lockley is regarded as the father of Ecology.

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[2] See our website article "Solar Powered Pterosaurs" or read my novel *Pterodactyl's Blood* , Amazon 2020